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PHOTOELASTIC ANALYSIS OF STRESS IN A DAM CONTAINING A LARGE GALLERY

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PHOTOELASTIC ANALYSIS OF STRESS IN A DAM CONTAINING A LARGE GALLERY

R. L. Sanks,[1] A. M. ASCE

SYNOPSIS

Mathematical analysis of a large gravity dam was made difficult—and perhaps impossible at the present state of the science—by the presence of a large gallery to house the turbines. Preliminary studies of two-dimensional models by photoelasticity determined dead and live load stresses with excellent accuracy. The techniques reported furnish a practical and accurate solution; construction of dams of this type need not be avoided because analytical analyses appear to be hopeless.

INTRODUCTION

As part of an enormous program to develop hydroelectric power on the Columbia River, the Corps of Engineers, Department of the Army, constructed Chief Joseph Dam about 50 miles downstream from Grand Coulee Dam. In 1943 several alternative designs were being studied, one of which, hereinafter called Foster Creek Dam, was unique in that it contained an immense gallery to house the turbines and generators.

A cross section of Foster Creek Dam (Fig. 1) shows a gallery about 65 feet high and 65 feet wide. It extends the full length of the main portion of the dam. The piers are 50 ft. on centers, and the turbines are 60 ft. on centers. Vertical sliding gates installed in the pier guides permit the water level to be raised to a maximum elevation of 229 ft. above the base of the dam.

The Problem

Most large dams contain galleries, but they are usually small in comparison to the size of the dam and cause only localized stresses. The gallery in Foster Creek Dam, however, is so large that it seriously affects the entire stress distribution. Accordingly, ordinary methods of analysis are not applicable for the solution of boundary stress or internal stress distribution. It is therefore necessary to resort to the more general mathematical method of "theory of elasticity" or to some experimental method in order to determine the stresses with a desirable degree of accuracy. However, the boundary conditions are so complex that a solution by theory of elasticity would be extremely difficult, and perhaps even impossible; if the third dimension is considered, the difficulties are multiplied many times, and a solution appears hopeless.

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Purpose

The purposes of this paper are to evaluate the photoelastic method as applied to this type of problem, to present the experimental techniques, to present the complete stress distribution caused by dead load and by live load, and to indicate the accuracy of the results and the application to the prototype.

Scope

The scope of the paper is limited to the solution of live and dead load stresses in the overflow section. Stresses caused by the intake and draft tubes are omitted. Although no suggestions are made for revision of the design, it is apparent that stresses can be reduced. The effects on stresses of temperature variations, shrinkage, growth, construction joints and pouring planes, flexible foundation, uplift due to seepage, torque on the turbine bases, vibration caused by the turbines and turbulence in the water, and pressures caused by high tail water and the nearly vertical force of water on the crest of the dam upstream from the sliding gates are not considered. The problem is assumed to be one of stresses acting only in the plane of the cross section.

A glossary of terms is given in the appendix.

Photoelastic Stress Analysis

While photoelasticity has come to be recognized as a standard method of experimental stress analysis, the "freezing" of gravitational stress patterns is not so widely known, and hence it has seemed desirable to include a brief history of its development.

The phenomenon of frozen stress patterns was discovered by Maxwell[5] in 1849. He observed that a jelly of isinglass subjected to a torque during the process of cooling and hardening caused a photoelastic effect even after the load was removed. However, he could not explain the phenomenon, and his work was forgotten until 1935.

In experiments with Marblette, Solakian[9] heated and annealed a circular rod under the action of an applied torque. Slices held in polarized light showed concentric circles which could be thought to represent shearing stress distribution. This single experiment, however, did not furnish proof of the universal application of the method.

Oppel[6] was the first to state that cutting a model did not affect the frozen stress pattern. He also found that the frozen stress pattern in a rectangular block subjected to a concentrated load agreed with results obtained by the theory of elasticity.

Hetenyi[3] proved beyond doubt that careful cutting of a model does not disturb the frozen stress pattern. He suggested the plausible diphase theory of the structure of Bakelite and similar plastics, thus explaining why frozen stress patterns can be obtained. He also proved that Bakelite BT 61-893 behaves elastically even at elevated temperatures. Moreover a twenty-fold increase in optical sensitivity occurs as it is heated from 20° to 110° C. Stress, strain, and fringe order remained mutually proportional up to a stress of 13 fringes in material 1/4 inches thick at 110° C.

In an effort to determine stresses due to gravitational forces Bucky, Solakian, and Baldin[2] placed a model in a centrifuge which in turn was placed in the field of view of a polariscope. A stroboscopic light source was flashed each time the model reached a position normal to the optical axis of the polariscope. To contain the wind storm the centrifuge was enclosed in a

metal case, and windows were provided to permit a view of the model. Although this experiment showed much ingenuity, it was not very practical. In addition to mechanical difficulties of obtaining good fringe photographs and isoclinic sketches from a model revolving at high speed, the investigators were hampered by the fact that Bakelite is relatively insensitive optically at room temperatures. Obtaining lateral strain measurements is certainly not feasible under such conditions.

Combining the work of Bucky, Solakian, Baldin and Hetenyi, Eberhart used a centrifugal oven to freeze centrifugal stresses into a Bakelite model of Hoover Dam. Photographs of the fringe pattern and sketches of the isoclinics were obtained in a polariscope, and lateral strains were obtained with a dial gage reading to 1/10,000-inches by surveying the thickness of the model before and after the freezing process. These lateral strains are reasonably accurate, because the modulus at high temperatures is low and poisson's ratio is high. The analysis of stresses in Hoover Dam was based on both the photoelastic and the lateral strain data. Since the difference of principal stress is given by the fringes, and the sum is given by the lateral strain,[10] the analysis is not subject to the usual errors involved in the graphical integration process commonly used where only optical data are obtained. Polivka[7] compared these results with the results of an analysis using only photoelastic data. His analysis[8]—more accurate than the usual graphical integration process—is based on the integration of the first derivatives of Airy's stress function. Polivka concluded that the centrifugal freezing method was valid because results agreed well with slab analogy experiments[11] on Hoover Dam. He stated that analysis based on lateral strain data is insufficiently accurate, especially at boundaries and near points of stress concentration, but his conclusion is somewhat contradicted by his curves, which show a reasonably good comparison of results.

After the tests reported herein were made, a new material, Fosterite, was discovered. Leven[4] reported it to be much superior to Bakelite for frozen stress patterns. It can be obtained in blocks of almost any size, machines freely, is elastic at high temperatures, and has a high modulus and a low fringe constant. The stress patterns are not affected by time. Furthermore the undesirable boundary condition of the stress patterns so common in Bakelite models (see Fig. 6) is completely absent in Fosterite models. Results from three-dimensional frozen-stress models show the most excellent agreement with theory of elasticity.

Even more recently, certain transparent epoxy resins have been discovered to be superior in some ways to Fosterite. These resins[2] can be cast by mixing a plasticizer with the vehicle.

Experimental Investigations

Experimental investigations of stress caused by dead load were begun at the University of California at Berkeley in September, 1943 under the supervision of Professor H. D. Eberhart. Two models were made and tested, one, herein called Model 1, represented the overflow section, while the other was a model of a pier section. Discussion of the latter has been omitted for brevity. Complete data were taken, but the only results worked out at that time were isostatics and boundary stresses. In 1949 the writer, working alone, revised the isostatics and boundary stresses and calculated the complete

2. C-4, C-6, and C-9 resins, Armstrong Products Co., Warsaw, Indiana.

stress distribution together with static checks and corrections for converting stresses from centrifugal to gravitational force fields.

Experimental work on the stress caused by live load—hydrostatic pressure—was begun in February, 1949 at the University of Utah. The model, called Model 2, represented the overflow section and was similar to Model 1. No studies of live load stress in a pier section have been made. As with Model 1, the laboratory data obtained consisted of fringe photographs, isoclinics, lateral strain surveys and calibration of the model material constants.

Of course, it was recognized that the problem was a three-dimensional one. It was, however, treated as a plane stress problem because it was deemed that the errors induced would not be excessive. (Three-dimensional problems in concrete are quite commonly treated as plane stress problems.) This was to be a preliminary investigation, and neither great accuracy nor exact similitude with the prototype was required. Furthermore, at that time there was no good model material obtainable in the size required for a suitable three-dimensional model.

Design of Models

In order to represent the prototype and yet permit easy manufacture of the models, the cross section finally adopted for Model 1 was chosen as shown in Fig. 2. It should be realized that at the stage of planning reached in 1943, little was known regarding the feasibility of this type of dam from a structural viewpoint, and therefore it was desirable to reproduce similitude in general rather than in absolute structural behavior.

Model 1 was not fixed at the base; however, in order to apply loads to Model 2, it was necessary to fix its base in a rigid clamp. The writer now feels that Model 1 should have been fixed at the base because: (1) the base of the prototype would be restrained by a foundation deliberately made rough; (2) Model 2 had to be made with a fixed base; and (3) since tests with gelatin models showed somewhat different stress patterns in the drown when the base was fixed rather than left free to slide, it is known that Models 1 and 2 are not strictly comparable in structural action.

A grid system was used to establish points of lateral strain measurement. In general, this grid system was satisfactory, but because a single value of lateral strain is not reliable, the grid system in the crown section should have been made much finer. The grid system of Model 2 was made similar to that of Model 1, but many sets of lateral strain measurements in Model 2 were taken at points other than at intersections of the grid.

Equipment and Apparatus

Polariscopes. Model 1 was tested in the polariscope in the Engineering Materials Laboratory of the University of California. This polariscope has an 8-inch diameter field, is optically well balanced, and is equipped with selsyn motors which rotate the polaroids in unison—a feature often indispensable for tracing dim isoclinics. Model 2 was tested in the University of Utah polariscope, an instrument with a field 11-1/2 inches in diameter and also equipped with selsyn motor drives on the polaroids.

Lateral extensometers. Lateral strains in Model 1 were measured by surveying the thickness at all grid intersections before and after the freezing process. The apparatus consisted of a Federal dial gage with a least reading of 0.0001-inches mounted on a heavy shaft rigidly fastened to a massive base. Contact points on the dial gage shaft and on the base were made of red lucite.

The least reading corresponds to 0.11 fringes of stress (0.4 psi, prototype) in Model 1.

Lateral strains in Model 2 were measured with a lateral extensometer built with features suggested by Weibel[13] and equipped with a Type F Huggenberger tensometer which has a multiplication ratio of 2000. The least reading of the tensometer is five millionths inches which corresponds to 0.15 fringes of stress (4 psi, prototype) in Model 2. The lateral strain was determined by reading the extensometer at no load, then full load, then no load on the model in rapid succession. Three sets of data were obtained at each point and the slightest variation in results was deemed sufficient to disqualify the data. In order to speed the loading and unloading of the model, a sylphon bellows was set under the loading lever. Compressed air piped to the bellows provided quick control of the loading lever without undesirable vibration.

Centrifugal oven. A gravitational force field was approximated by freezing centrifugal stresses into Model 1 in a centrifugal oven, which consisted of a welded steel frame about 31 inches long and 11 inches high fastened to a central shaft held in a large circular tank which was equipped with electrical heating elements controlled by a Micromax unit. The model was mounted in the rotor and enclosed between brass plates to protect it from air turbulence. The calibration piece was mounted at the opposite end of the frame and was also enclosed between brass plates. After static balancing, the shaft with its frame containing the models was set in its bearings in the tank and rotated at 910 rpm by a 1/2 hp motor. Small adjustments in speed were obtained by changing the tension on the vee belt. The temperature was increased to 118° C in a 2-1/2 hr. period, held constant for 3-1/2 hours, then decreased to 28° C in a 16-hr. period. During this time the speed was maintained at 910 rpm, which resulted in an average centrifugal force of about 280 gravities at the center of the model. Since air friction prevented cooling the model to room temperature, dry ice was dropped into the oven through ports provided for that purpose.

Water loading device. The device used to apply a load simulating hydrostatic pressure consisted of eight steel pads, which were in essence "reactions" for four short beams; each beam was a "reaction" for two more beams, which in turn were "reactions" for a single beam acted upon by a concentrated load. Thus a single load can by such a "stack" of beams be separated into eight parts, each part being any desired proportion of the whole. Fulcrum points were carefully machined and hardened so that the pressure at the top was equivalent to 25.6 psi and, at the bottom to 99.2 psi (prototype values). This loading corresponds to high water 229 feet above the base (See Fig. 1). Water load on the crest of the model was ignored; it would be troublesome though not impossible, to load the crest of the dam. By placing the pads upon a 26 gage strip of brass separated from the model by two strips cut from thick blotters, stress concentrations were prevented.

The device worked very well. The actual pressure distribution corresponded to the ideal pressure distribution within an average error of three per cent. Maximum error was seven per cent.

Testing

The models were made of Bakelite BT 61-893 annealed, ground, polished and machined in the usual way by hand. Dimensions are shown in Figs. 2 and 3.

Calibration. Each calibration specimen (Figs. 2 and 3) was subjected to precisely the same treatment as its model. Model 1 calibration specimen was tested in tension, because that was the only safe way to load it. Model 2 calibration specimen was tested in compression. By filing or scraping the gabled ends, the bearing point can be positioned at will. It is possible to obtain perfect axial loading very easily and quickly.

Calibration curves are shown in Figs. 4 and 5. The size of each circle representing a plotted "point" indicates the error of reading.

Model 1. The model was carefully surveyed with the 0.0001-inch dial gage before and after stresses were frozen into it. The differences were reduced to stress in terms of fringes using the calibration curve of Fig. 4b. Contours of these stresses (sum of principal stresses) are plotted in Fig. 6b, Isopachics, in order to correspond with the fringes of Fig. 6a (difference of principal stresses). The isoclinics of Fig. 6c were obtained by means of sketches using plane polarized white light in the polariscope. They show the angle between a principal stress and the vertical reference axis. The isostatics, or stress trajectories, in Fig. 6d were obtained by plotting the data of Fig. 6c; they show graphically the direction of each principal stress.

Because the model was subjected to a varying centrifugal force field while the prototype would be subjected to a uniform gravitational force field, it was necessary to correct centrifugal to gravitational stresses. The method used for the correction assumes that the directions of stress in centrifugal and gravity force fields are similar and that the true stresses on any section are such that the vertical stress integrated over the cross-sectional area equals the known vertical force acting on the section. The stresses on different sections must, of course, be multiplied by different factors.

The vertical forces acting on horizontal sections of both the model and a one-inch-thick slice of the prototype are shown in Table 1. The unit weight of concrete was assumed to be 145 lb per cu ft. The weight of Bakelite was found to be 0.0498 lb per cu in. The similitude relationships are shown in the last column of Table 1b. It should be noted that these relationships were computed from data on the speed and radius of rotation, the density of Bakelite, and the size of the model; none of the photoelastic data was used in any way.

Knowing the difference of principal stresses from the fringe photograph (Fig. 6a) and the sum of the principal stresses from the isopachics (Fig. 6b), each principal stress can be obtained by arithmetic. With each principal stress determined, and their directions known from the isoclinic map and isostatic sketch, shear and normal stresses on any plane can be computed by use of equations of mechanics or by consideration of Mohr's circle. Shear and normal stresses for Model 1 are shown in Fig. 7.

Bakelite BT 61-893 is not an entirely suitable material for frozen stresses, although in 1943 it was the best available. Its most noticeable defect is a time-edge effect; a sudden shift of the fringe occurs within about 0.05 inches of the boundary. The sudden shift was ignored in the analysis of stress by extrapolating internal data to the boundary.

In order to determine the accuracy of these stresses, static checks were made comparing known external forces to internal forces found by integrating the stress over the cross-sectional area. These checks are shown in Table 2.

Based on the experience of interpreting data, it appears that the error of plotting fringes is about 0.05 fringes (except at the boundaries, where the error is probably 0.10 fringes); the error of plotting isopachics is about 0.25 fringes. It is felt that the position of the isoclinics is correct to within 0.05 inches except in the region bounded by coordinates E, G, 7, 9 and J, L, 8, 12,

where the error may be greater because the isoclinics were hard to see. In summary it seems that the maximum error is about 0.25 fringes or 12 psi in prototype stress. This estimated maximum error compares favorably with the maximum average error of the static check, 6.1 prototype psi.

When centrifugal forces are converted to gravity forces, the above errors are increased, because the conversion is only approximate. Since the maximum difference between centrifugal and gravity forces was only plus or minus 11 percent, the error at the base, even if no correction were applied, would be only 8 prototype psi, which added to the 12 psi error above would give a total maximum error of 20 psi. Because the correction is certainly a step in the right direction, it might reasonably be assumed that the corrected error at the base would be less than 16 prototype psi. At locations above the base the errors would be less.

Model 2. The data obtained from Model 2 is shown in Fig. 8. When the isopachics were first plotted it appeared that some of the data were inaccurate. Large errors can be detected by visualizing a membrane (soap film, for example) tightly stretched between the boundaries of the model. If the membrane is supported only at the boundaries and if the height at the boundaries is proportional to the sum of principal stresses, then its height at all points will be proportional to the sum of principal stresses.(12) Therefore the isopachic data must plot as smooth curves since the membrane would naturally take the form of a smooth, warped, surface. Wherever inaccuracies were suspected, the entire area was resurveyed with the lateral extensometer; lateral strains were often taken at points between grid intersections. Most of the original data were found to be correct; suspected errors were usually the fault of interpolation between plotted points. The extreme care used in obtaining, plotting, and checking lateral strain data accounts for the unusual accuracy of the analysis.

Shear and normal stresses were computed as described for Model 1. These data are shown in Fig. 9. Static checks, comparing known external forces to integrated internal stresses are shown in Table 3.

Based on experience in gathering and interpreting data it is felt that the error of plotting fringes is about 0.05 fringes, and the error of plotting isopachics is 0.15 fringes. The maximum probable error is therefore about 0.2 fringes or 5 psi in prototype stress. This estimated maximum error compares with the maximum average error of 1.4 psi in Table 3.

Similitude. Similitude relations between Models 1 and 2 and their prototypes are very simple. Stress distribution is the same in each, and therefore it is only necessary to determine the ratio between any homologous stresses in model and prototype. All other stresses bear the same relationship.

SUMMARY AND DISCUSSION

No curve has been altered in any way after a static check was made; no effort has been made to modify the curves to obtain better static checks. Therefore the static checks reflect the inherent accuracy of the photoelastic method. The poorest static check in Model 1 was 6.1 psi (prototype stress); in Model 2, 1.4 psi. The maximum probable error in Model 1 is estimated to be 12 psi (prototype stress); in Model 2, 5 psi. Compared to the strength of concrete, the errors are so small as to be almost unbelievable.

Although the accuracy of the results for each model is excellent, the

models themselves do not faithfully represent the prototype; consequently, the results of these experiments are not directly applicable to the prototype. Discrepancies between model and prototype are as follows: (1) A three-dimensional prototype is represented by a two-dimensional model. For a strictly representative solution the third dimension cannot be ignored. (2) The base of Model 1 was not fixed, and therefore stresses obtained from it are somewhat in error, particularly at the crown. (3) Some liberties were taken with the prototype dimensions, chiefly in rounding the corners of the large gallery and omitting the small gallery. This results in smaller stress concentrations. Because concrete is a plastic material, stress concentrations can be largely ignored as shown by the experiments of Blanks and McHenry.[1]

None of the above discrepancies is inherent in the photoelastic method. Three-dimensional models of Fosterite or epoxy resin can be machined or cast; live load, dead load, or both can be frozen into the material, and the model can be sliced into thin sections for analysis. Since the epoxy resins are insoluble in most chemicals, it should be possible to cast a model using soluble cores for galleries and draft tubes, dissolving the cores after curing the castings. To be representative, a monolithic model, restrained at the ends and base, should include a pier section and intake and draft tubes. The analysis of three-dimensional models is much more complex and time consuming than that for two-dimensional ones. Nevertheless, suitable techniques have been developed, and good results can be obtained.[14] A three-dimensional analysis is, therefore, perfectly feasible. It is hardly necessary to state that fixing the base of Model 1 or making the model dimensions correspond closely to those of the prototype would have been easy to do. Any misrepresentation of the prototype is the fault of the model, then, and not of the photoelastic method; photoelasticity has been shown to be exceptionally accurate, and it is a practical method of analysis for stresses in a dam as well.

Still, there are conditions and stresses within the prototype which a photoelastic model cannot represent; for example: pouring planes, construction planes, cracks, thermal effects, plastic flow, growth, seepage, and vibration from turbines. The effects of these factors cannot be determined by mathematics either (with any certainty or accuracy); so, where the photoelastic method breaks down, all other methods (except full scale testing of the prototype) do also.

Notwithstanding the faults of similitude of the models, the results obtained are sufficient for a preliminary analysis, which was the purpose of the tests. By inspecting Figs. 7 and 9, it is evident that there are regions of tension in the model when loaded by either gravity or hydrostatic pressure. When hydrostatic stresses are added to gravitational stresses, tension is confined to two small areas near D-8 and H-7; the maximum is 12 psi (prototype stress). The dam should be redesigned to eliminate tension due to gravity plus hydrostatic loads and to reduce tension due to gravity loads before starting a more rigorous program of tests of three-dimensional models.

The equipment and apparatus used to test the models was quite satisfactory except for the Federal dial gage device used to survey the thickness of Model 1. An error in reading cannot be checked after stresses are frozen or annealed, and the inherent accuracy is not high. A precision profiling machine would be far better. The extensometer fitted with the Huggenberger Type F tensometer, although often temperamental, gave good results. Undoubtedly, a Statham transducer[3] fitted to the lateral extensometer would be

3. Statham Laboratories, 12401 West Olympic Blvd., Los Angeles 64, Calif.

much superior to the Huggenberger tensometer. Other investigators[7, 14] have reported poor results with extensometers set near boundaries. Lateral strain data taken 0.02 inches from boundaries of Model 2 and 0.08 inches from boundaries of Model 1 were excellent except where the stress gradients were very high or where the boundary was sharply curved.

CONCLUSIONS

1. Stresses in the two-dimensional models were determined with excellent accuracy—more than sufficient for engineering analysis.
2. Discovery of new photoelastic model materials insures that even better results can be obtained with frozen stress models.
3. Considering the amount of engineering involved in the building of a dam, the time required to obtain and reduce the photoelastic data is not excessive.
4. Photoelasticity is a practical method for solving stresses in dams and similar structures when mathematical analysis appears hopeless or impractical.
5. The techniques were satisfactory. Lateral strain measurements near boundaries were found to be reliable except at points of high stress gradient or where boundaries are sharply curved. Lateral strains in frozen-stress models were unreliable where fringes were unreliable; that is, within 0.05 inches of the boundary. This unreliability of fringes and lateral strain measurements is due to a "time-edge" effect peculiar to Bakelite; it does not occur in some of the newer materials. Errors due to substituting a centrifugal force field for a gravitational force field are small.
6. Stresses obtained from two-dimensional models are applicable to the prototype (except in regions close to draft and intake tubes and piers) with sufficient accuracy for preliminary design. Stresses near piers and intake and draft tubes as well as stresses for the final design would have to be determined from three-dimensional models, which would have to include at least one pier and a set of intake and draft tubes.

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APPENDIX

Fringe - A black band, appearing in a model strained in a field of

monochromatic polarized light, which is the locus of all points of some particular value of the difference of principal stresses.

Fringes - Stress expressed in terms of the fringe value.

Fringe value - The stress in psi required to produce one fringe in a model.

Fringe constant - The stress in psi required to produce one fringe in model material one inch thick; fringe value per inch. For Bakelite BT 61-893, it varies from 82 to 87 psi/in.

Isoclinic - A dark band which appears in a strained model in a field of plane polarized light. It is the locus of all points at which the direction of principal stress corresponds to the axis of polarization.

Isopachic - The locus of all points of some particular value of the sum of the principal stresses.

Isostatic - Also "stress trajectory." A line oriented in the direction of a principal stress.

S_u - The major principal stress.

S_v - The minor principal stress.

S_x - A normal stress acting in the horizontal direction. Compressive stresses are plotted downward.

S_y - A normal stress acting vertically. Compressive stresses are plotted downward.

S_{xy} - A shearing stress acting on horizontal and vertical planes. Stresses acting down on the left side of a free-body element are considered negative and plotted downward.

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TABLE 1.

CONVERSION FACTORS FOR CENTRIFUGAL TO GRAVITATIONAL STRESS--MODEL 1.

(a) Forces on Segments

Segment	Model			
	Area, Sq. in.	Centrifuge Radius to Centroid, in.	Centrifugal Force, gravities	Load per Segment, lb.
Top-D	0.40	10.16	239	1.20
D-E	1.09	10.48	237	3.39
E-F	0.84	10.94	258	2.74
F-G	0.90	11.38	268	3.03
G-H	1.18	11.85	279	4.16
H-I	1.66	12.27	289	6.05
I-J	1.97	12.69	298	7.39
J-K	2.15	13.13	309	8.37
K-L	2.15	13.57	319	8.65
L-Base	2.19	14.02	330	9.11

(b) Stresses on Sections

Section	Model		Prototype		Ratio, Prototype to Model Stress, S_y, Prototype / S_y, Model
	Total Load, lb.	Average Vertical Stress, psi	Total Load 1-in. Slice, lb.	Average Vertical Stress, psi	
D	1.20	2.1	9,800	8.0	3.82
E	4.59	9.8	36,500	36.3	3.73
F	7.33	14.3	57,100	52.5	3.67
G	10.36	18.7	79,100	67.0	3.58
H	14.52	14.5	108,000	50.7	3.50
I	20.58	19.2	149,000	65.0	3.39
J	27.96	22.2	197,000	73.2	3.30
K	36.33	28.8	249,000	92.5	3.21
L	44.98	35.7	302,000	112.0	3.14
Base	54.09	43.0	356,000	132.0	3.07

TABLE 2.

STATIC CHECKS. MODEL 1.

Section	Normal Force		Average Error	
	External (from Table 1), lb.	Internal (from photo-elastic results Fig. 7), lb.	Model, psi	Prototype, psi
D	1.20	2.10	1.6	6.1
E	4.59	4.5	0.2	0.7
F	7.33	6.5	1.6	5.9
G	10.36	10.1	0.7	2.5
H	14.52	14.7	0.2	0.7
I	20.57	21.0	0.4	1.4
J	27.96	28.8	0.6	2.0
K	36.33	25.5	0.6	1.9
L	44.98	44.1	0.7	2.2

TABLE 3.

STATIC CHECKS. MODEL 2.

Sec-tion	Shear Force				Normal force		
	External, lb.	Internal, from Photoelastic Results, Fig. 9	Average Error Model, Psi	Average Error Proto-type, psi	Internal, lb.	Average Error Model, psi	Average Error Proto-type, psi
D	0	2	5	0.4	5	11	0.8
E	17	17	0	0.0	5	15	1.1
F	57	56	3	0.2	4	10	0.8
G	107	100	16	1.2	8	19	1.4
H	166	161	7	0.5	14	18	1.4
I	232	217	18	1.4	5	6	0.5
J	312	313	1	0.1	5	5	0.4
K	399	400	1	0.1	7	7	0.5
L	496	490	6	0.5	8	8	0.6

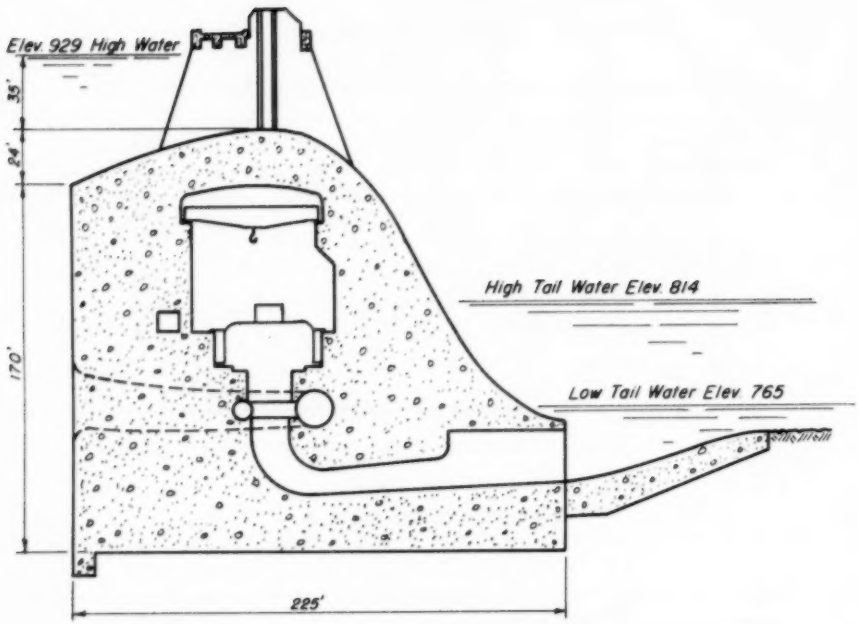
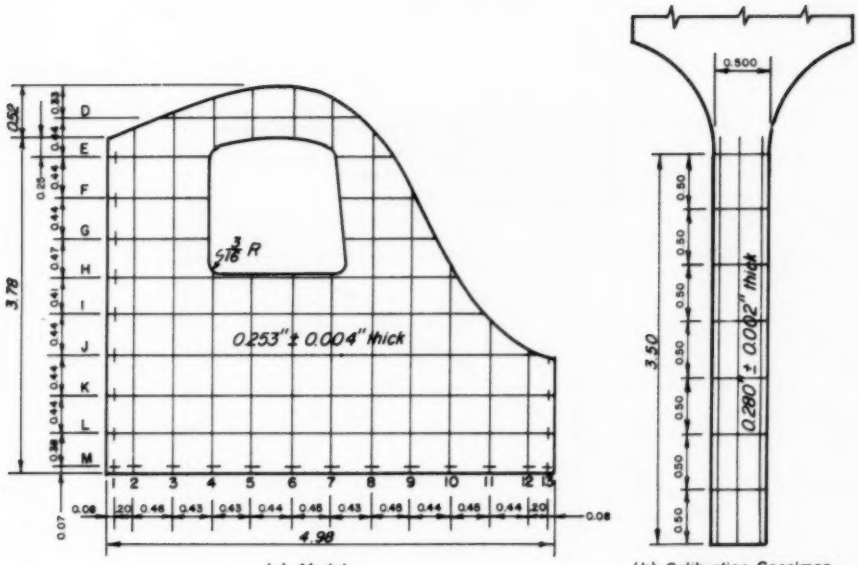
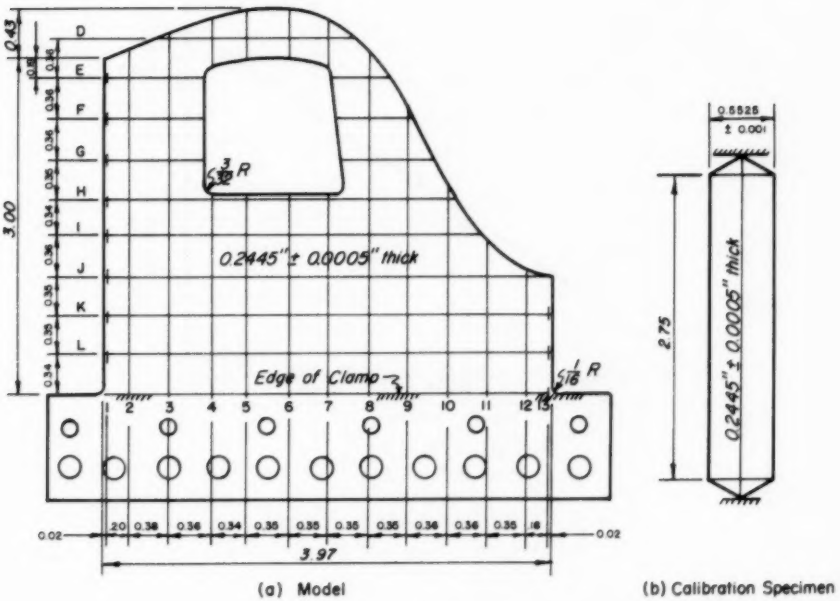


FIG. 1 FOSTER CREEK DAM
Section through Draft Tube



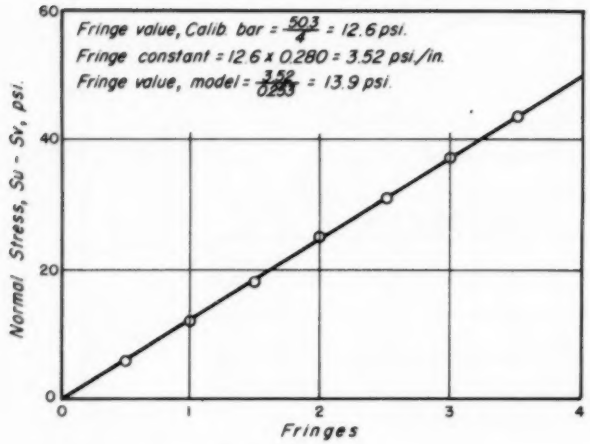
(a) Model (b) Calibration Specimen

FIG. 2. MODEL 1.

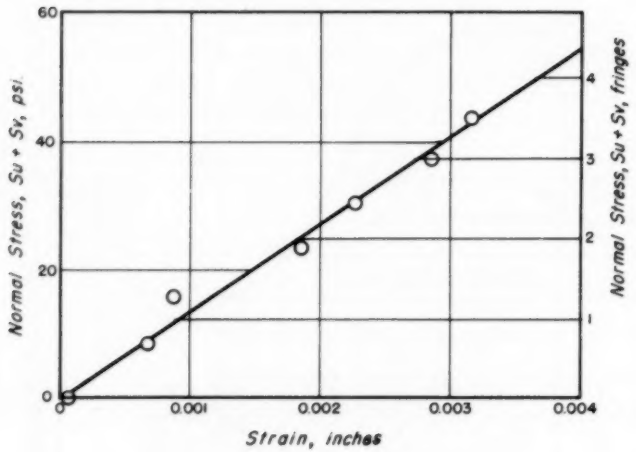


(a) Model (b) Calibration Specimen

FIG. 3 MODEL 2

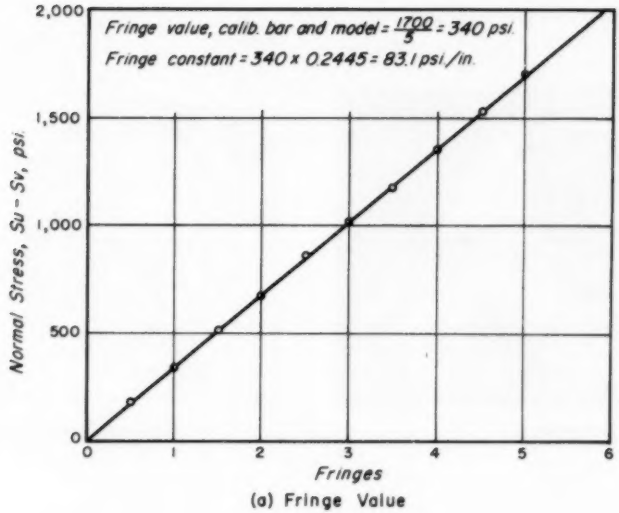


(a) Fringe Value

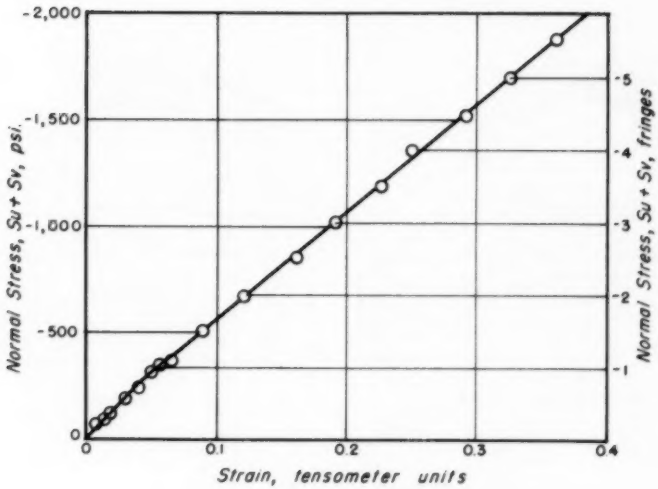


(b) Lateral Strain

FIG. 4. CALIBRATION CURVES. MODEL I.

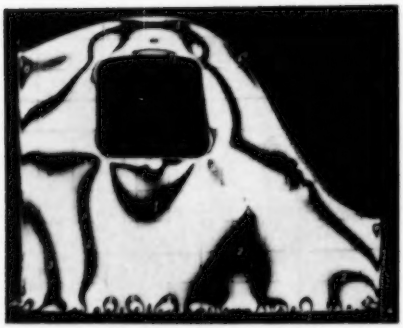


(a) Fringe Value

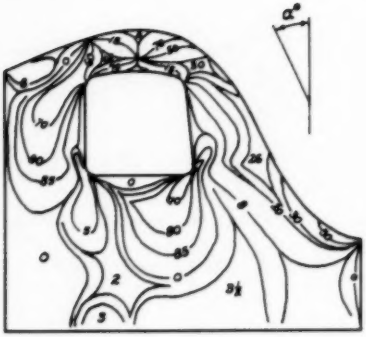


(b) Lateral Strain

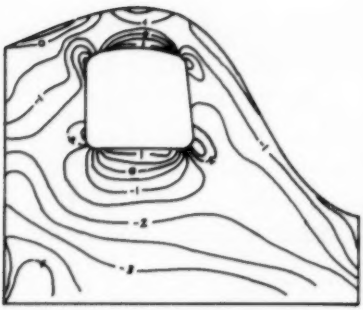
FIG 5 CALIBRATION CURVES MODEL 2



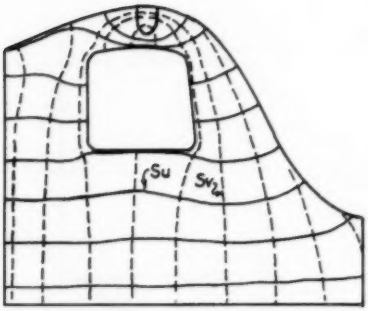
(a) Fringe Photograph



(c) Isoclinics, α°



(b) Isopachics, fringes



(d) Isostatics

FIG 6. SUMMARY OF DATA. MODEL I.
CENTRIFUGAL FORCE FIELD

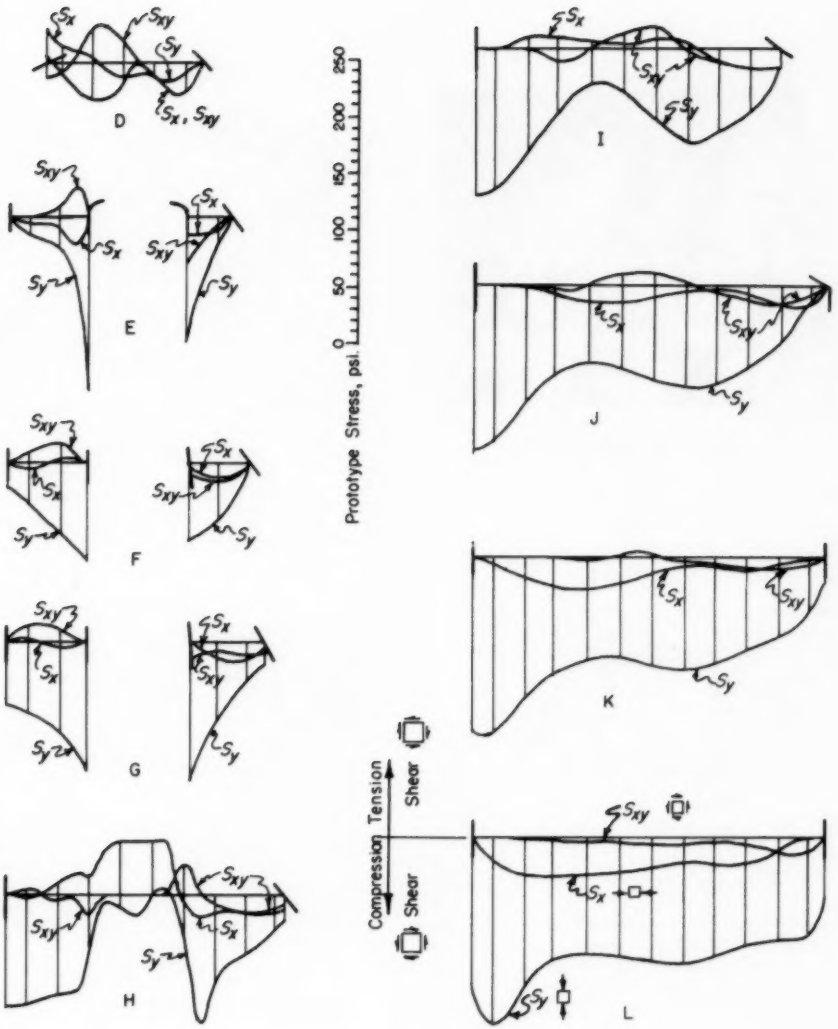
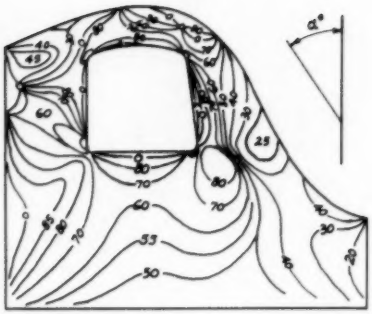
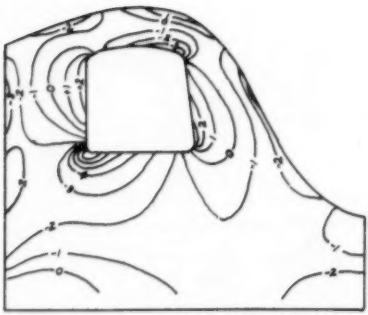


FIG. 7 SHEAR AND NORMAL STRESSES. MODEL I.
GRAVITY FORCE FIELD

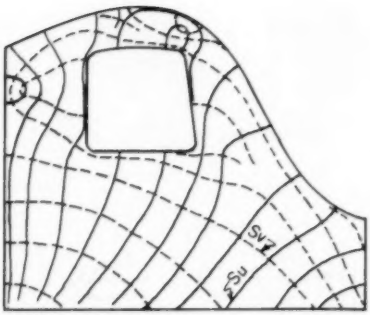
(a) Fringe Photograph



(c) Isoclinics, α°



(b) Isopachics, fringes



(d) Isostatics

FIG. 8. SUMMARY OF DATA. MODEL 2.
HYDROSTATIC PRESSURE ONLY

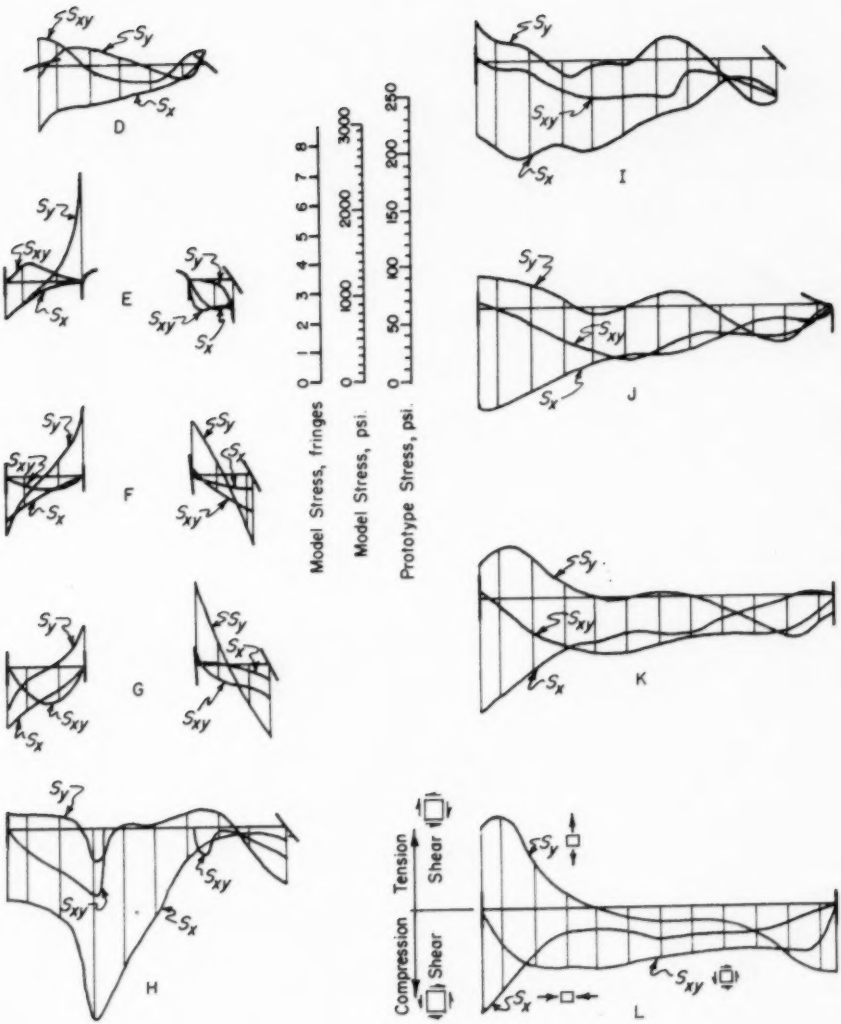


FIG. 9 SHEAR AND NORMAL STRESSES. MODEL 2.
HYDROSTATIC PRESSURE ONLY

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